A Discrete Time Markov Chain Model using Maximum likelihood for the Assessment of Inflation Rate in Pakistan

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Abstract

Markov chains epitomize a class of stochastic process for a wide range of applications. Specifically, discrete time Markov chains (DTMC) is employed to model the transition probabilities between discrete states with the help of the matrices. To examine and forecast the time series the Markov chain model is applied. The most important indicator in macroeconomics is inflation, which persisted in double digits in 1970s and also in last several years. Different states are checked with the model by using inflation rate data form July 2000 to April 2015. A simulation technique is used for random sequences of inflation predefine states for one year and take 1st quarter data from it and then model the estimates by maximum likelihood then checked the equilibrium distribution.

Key words: Inflation; Markov Chain; Maximum Likelihood; Time Series

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1. Introduction

A Markov chain is a stochastic process with the MP. The term "MC" refers to the arrangement of arbitrary variables such a procedure exchanges through, with the MP significant sequential dependence only between neighboring periods. This system describes a chain of linked events, in which coming situation depends on current situation of the system. Different types of MP are selected as "MC" in the previous literature. Generally, the stretch is kept for a process with a discrete set of times is called discrete time Markov chain (Everitt, B.S. 2002). Discrete Markov chain models are valuable for demonstrating mostly applied structures such as trade systems, line up systems (Ching, 2001) and account systems (Chinget al., 2003). These models are used for displaying definite data series can also be found in mostly real world areas (Chinget al., 2002). Time series are used often in mostly real world areas. If anyone modeled the time series precisely, then it is easy to make accurate estimates and also optimally forecast in a decision procedure (Chinget al., 2008). A DTMC is an arbitrary procedure that undertakes changes from one situation to another situation. It must hold a "memorylessly", in which the chances of the next state depend on the current state not on the system of events that goes before. This case of the Memorylessness is called Markov property (MP). In real world processes; statistical models MC have many applications. MC refers to a continuous deprived of explicit indication. Although the time is taken as a discrete parameter, there is no restriction on the state space in the MC, the period may mention to a method on a random state space (Meyn, S. et al., 2009). Although in MC applications finite or infinite states are considered as discrete state spaces that have more statistical analysis, moreover time catalogue and state space restrictions, a lot of disparities additions and simplifications are founded.

In economics, inflation is a continued fluctuation in the general price level of goods and services in an economy with respect to a period of time. Inflation delivers important vision on the national economy and occurs in any economy but with a different rate and strength. Inflation in the price of food is a biggest issue being handled by emerging states a like Pakistan. This indicator use excessive quantity of compression with the economic circumstances of any state (Joiya &Shahzad, 2013). The movable economic and financial policies of the government of Pakistan have caused in upgrading in numerous macroeconomic factors with Gross Domestic Product (GDP) growth in several years. For example in 1960s, 1980s and the few centuries of the first era of the 21st century, this persisted above 6 percent during 2004-06. Despite this impressive performance of the economy, some worrisome factors have also seemed on this section. The most substantial of these indicators is inflation, which persisted in dual number in 70s and also in last numerous years (Ahmed et al., 2014).

2. Methodology

The expansionary or movable economic and fiscal strategies of the government of Pakistan have caused in development in numerous macroeconomic indicators containing Gross Domestic Product (GDP) growth in numerous years. For example in 1960s, 1980s and the scarce years of the first era of the 21st century, this continued above 6 percent throughout 2004-06. Although this inspiring presentation of the economy, some troublesome indicators have also seemed on the scene. The most important of these factors is inflation, which endured in double digit in 70s and also in previous numerous years. It has been founded that numerous supply and demand side indicators do contribute to intensification in inflation. A high progression in M2 and loose credit policy was also the cause of high inflation in the country (Khan and Axel, 2006). For checking the variability in the inflation rates, the inflation rate data is taken on monthly basis form July-2000 to April-2015 from state bank of Pakistan. Then separate inflation rate into 3 states, if rate<=0 then it categorize as “Deflation”, if <=1 then it categorize as “Creep”, if >1define as “Normal” inflation. A Discrete Time Markov Chain

(DTMC) is a sequence of random variables$X_1, X_2, X_3, \dots\ X_n$, characterized by the Markov property. The Markov property states that the distribution of the forthcoming state X_{n+1} depends only on the current state X_n and doesn't depend on the previous ones$X_{n-1}, X_{n-2}, \dots, X_1$.

$$P\left(X_{n+1} = x_{n+1}/X_1 = x_1, X_2 = x_2, \dots, X_n = x_n\right) = P\left(X_{n+1} = x_{n+1}/X_n = x_n\right) \quad (1)$$

The set of possible states $S = \{S_1, S_2, S_3, \dots, S_n\}$ of X_n can be finite or countable and it is named the state space of the chain (Ching.Wet al., 2008). Firstly build a Markov chain model for a pragmatic definite data sequence, adopt the following canonical form representation:

$$X_0 = (0,1,0)^T X_1 = (1,0,0)^T X_2 = (0,0,1)^T \ \dots \quad X_n = (0,1,0)^T$$

For $X_0=2, X_1=1, X_2=3 \dots X_n=2$.

To estimate the transition probability matrix for the above perceived Markov chain, considered the following simple measures. By totaling the transition frequency from State into State jin the arrangement, build the transition frequency matrix N then the transition probability matrix P for the arrangement as follows:

$$P = \begin{bmatrix} \frac{n_{11}}{n_{1j}} & \frac{n_{12}}{n_{1j}} & \frac{n_{13}}{n_{1j}} \\ \frac{n_{21}}{n_{2j}} & \frac{n_{22}}{n_{2j}} & \frac{n_{23}}{n_{2j}} \\ \frac{n_{31}}{n_{3j}} & \frac{n_{32}}{n_{3j}} & \frac{n_{33}}{n_{3j}} \end{bmatrix}$$

After making the transition matrix check the distribution of the states by using different initial states and check the stability situation after n generation of distribution of the states. In other words to find out the stationary distribution and identifying absorbing and transient states. For statistical analysis simulating a random sequenc e from an underlying DTMC. Then checks the estimates by using two methods, maximum likelihood.

The maximum likelihood estimator (MLE) of the p_{ij} entry, where the n_{ij} element consists in the number sequences ($X_t = S_i$; $Xt+1 = S_j$) found in the sample, that is

$$\hat{p}^{MLE}{}_{ij} = \frac{n_{ij}}{\sum_{u=1}^{k} n_{iu}} \qquad (2)$$

3. Empirical Results

3.1. Transition Probability Matrix

Table 1: Probability Matrix

	CREEPING	TROTTING	GALLOPING
CREEPING	0.4823529	0.3176471	0.2000000
TROTTING	0.4137931	0.4137931	0.1724138
GALLOPING	0.5588235	0.2352941	0.2058824

This matrix show that the probability of inflation rate from creeping to creeping is 48.23%, 20% creeping to galloping and 31.76% of creeping to trotting. Similarly the probabilities of inflation rate going from galloping to creeping is 55.88%, 23.52% from galloping to trotting, and 20.58% from galloping to galloping. The probability inflation rate of going from trotting to creeping is 41.37%; from trotting to trotting is 41.37%, and 17.24% from trotting to galloping inflation.

3.2. Igraph of Transition Probability Matrix:

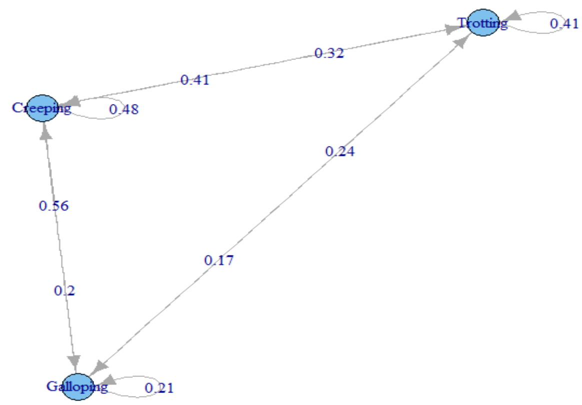

Figure 1: Igraph of Transition Probability Matrix:

For the graphical presentation of transition matrix in Markov chain model I graph is used, now according to the graph we can easily explain the probability of inflation rate moves from one state to another state. Let Creeping = C, Galloping = G, Trotting = T. this graph shows that the occurrence of inflation rate from C to C is 48%, 20% creeping to galloping and 32% creeping to trotting. The probability of G to C is 56% G to G is 21% and 24% chances of inflation rate from G to T inflation. Similarly the chances of inflation rate from T to C is 41% , 17% from T to G and 41% chance of inflation rate moves from T to T.

3.3. Equilibrium Distribution of Transition Matrix:

First chose [1, 0, 0] this initial state in which 1st state of inflation rate is present others are absent then calculate its initial probability vector which is X_0 =[0.4823529, 0.2000 0.3176471]. This is also called the generation of probability vector which explains that there are 48.23% probability of creeping state and 20% probability of Galloping and 31.76% probability of Trotting state. The entire probability vectors are generated by $X_0 * P^n$ this formula.

After checking the Nth generations

Table2: Equilibrium distribution

AFTER GENERATION N	CREEPING	GALLOPING	TROTTING
0	0.4823529	0.2	0.3176471
1	0.4758692	0.1924138	0.331717
2	0.4743244	0.191981	0.3336945
3	0.4741558	0.1919239	0.3339203
4	0.4741359	0.1919174	0.3339467
5	0.4741336	0.1919166	0.3339498
6	0.4741333	0.1919165	0.3339501
7	0.4741333	0.1919165	0.3339502
8	0.4741333	0.1919165	0.3339502
9	0.4741333	0.1919165	0.3339502

By using this initial probability vector generates next probability vectors and checks the stability in which generation inflation states probability vectors are stable. After 7th generation the probability vector of inflation states shows the stable chances. We can see it from the above table the probability vectors of 7th, 8th and 9th generation are stable at specific probability vector, which explain that there are 47.41% chance of Creeping, 19.19% of Galloping and 33.39% of Trotting state. All the next generations will show the same chances of occurrences of the inflation states, we may generate it into 10th, 11th times or so on many times.

3.4. Conditional Distributions:

Conditional distribution is used for checking the probability of different states of inflation rate if any one of state is given. In the conditional distribution of inflation states, given that current inflation state is Trotting.

Table 3: Conditional distribution for Trotting

CREEPING	GALLOPING	TROTTING
0.5588235	0.2352941	0.2058824

There are 55.88% chances of Creeping, 23.52%chances of Galloping and 20.58% of Galloping when current state is Trotting.

The conditional distribution of inflation states, given that current inflation state is Galloping.

Table 4: Conditional distribution for Galloping

CREEPING	GALLOPING	TROTTING
0.4137931	0.4137931	0.1724138

There are 41.37% chances of Creeping, 41.37 chances of Galloping and 17.24% of normal when current state is Trotting.

The conditional distribution of inflation states, given that current inflation state is Creeping.

Table 5 Conditional distribution for Creeping.

CREEPING	GALLOPING	TROTTING
0.4823529	0.3176471	0.200000

There are 48.23% chances of Creeping, 31.7% chances of Galloping and 20% of Galloping when current state is Trotting.

3.5. Steady state:

If the MC is a time regular MC, then procedure is defined by a lone, matrix with independent time, p_{ij} then the vector π is called a stationary distribution. No assumption available for starting distribution; the chain meets to the stationary distribution paying little respect to where it starts. Such π is called the equilibrium probability distribution of the Markov chain.

Here the stationary distribution for the states of inflation is

$$\pi = [0.4741333, 0.1919165. 0.3339502]$$

This vector explains that there are 47.41% chance of Creeping, 19.19% of Galloping and 33.39% of Trotting state.

Maximum Likelihood estimation:

MLE Transition Matrix

Table-6: MLE Probability Matrix

	CREEPING	GALLOPING	TROTTING
CREEPING	0.4736842	0.1842105	0.3421053
GALLOPING	0.4805195	0.2727273	0.2467532
TROTTING	0.3111111	0.2074074	0.4814815

This transition matrix show that the occurrence of inflation rate from Creeping to Creeping is 47.36%, 18.42% Creeping to Galloping and 34.21% Creeping to Trotting. The probability of Galloping to Creeping is 48.05%, Galloping to Galloping is 27.27% and 24.67%chances of inflation rate from Galloping to Trotting inflation rate. Similarly the chances of inflation rate from Trotting to Creeping is 31.11%, 20.74% from Trotting to Galloping and 48.14% chance of inflation rate moves from Trotting to Trotting inflation rate.

3.6. Equilibrium Distribution of MLE:

First chose [1, 0, 0] this initial state in which 1st state of inflation rate is present others are absent then calculate its initial probability vector which is X_0 =[0.4736842, 0.3421053 0.1842105]. This is also called the generation of probability vector which explains that there are 47.36% probability of Creeping state and 18.42% probability of Trotting and 27.41% probability of Galloping state. The entire probability vectors are generated by $X_0 * P^n$ this formula.

After checking the Nth generations

Table7: Equilibrium distribution for MLE

AFTER GENERATION N	CREEPING	GALLOPING	TROTTING
0	0.4736842	0.3421053	0.1842105
1	0.4193262	0.3722218	0.208452
2	0.4763518	0.1926844	0.3309639
3	0.4743968	0.1920034	0.3335998
4	0.474164	0.1919267	0.3339093
5	0.4741369	0.1919177	0.3339454
6	0.4741337	0.1919167	0.3339496
7	0.4741334	0.1919165	0.3339501
8	0.4741333	0.1919165	0.3339502
9	0.4741333	0.1919165	0.3339502
10	0.4741333	0.1919165	0.3339502

By using this initial probability vector generates next probability vectors and checks the stability in which generation inflation states probability vectors are stable. After 7th generation the probability vector of inflation states shows the stable chances. We can see it from the Table-7 the probability vectors of 8th, 9th and 10th generation are stable at specific probability vector, which explain that there are 47.41% chance of Creeping, 19.19% of Galloping and 33.39% of Trotting state. All the next generations will show the same chances of occurrences of the inflation states, we may generate it into 11th, 12th times or so on many times.

4. Conclusion

The current study based on Markov Chain Model, core purpose of present study was according to Markov Chain Model related to the economics features. For this study Macroeconomic indicator inflation rate is used and found that it is changing since several years. It has a foremost outcome on the progress and economic development of any country. Study data was form July 2000 to April 2015. we categorized our data into three different states that are "Trotting", "Creeping" and "Galloping" inflation rate as per as the key features of Markov

Chain Model. First of all Checked the Transition count and Transition Matrix where it is easily to check the progress chances of one state in comparison to another state. Equilibrium distribution is very important for the Markovian Transition Matrix which clarifies the stability point for the model whether it is checked on different initial vectors. Conditional distribution explained about the chances of other states if anyone is fixed that we know about it already then check the chances of there. The “markov chain” I graph R software package that helped for the Graphical presentation of the model. Equilibrium distribution is obtain of the model separately by mutually techniques and checks the stability point of these distributions by taking different initial vectors.

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